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Our efforts have continued on development of efficient accurate integration of Fokker Planck equations. Our objective is to analyze and optimize the dynamic behavior of nonlinear stochastic differential equations, especially the stochastic resonance effects based on the probability density function generated by the Fokker Planck equation. Our approach uses the backward characteristic method and the time-splitting integration of the Fokker Planck equation.

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FOKKER-PLANCK EQUATIONS: UNCERTAINTY IN NETWORK SECURITY GAMES AND INFORMATION

AFOSR-FA9550-09-1-0226
March 1, 2009-November 30, 2011

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Abstract
We have significant accomplishments on uncertainty quantification in inverse problems for dynamical systems, generalized sensitivity and optimal design of experiments, elasticity and viscoelasticity modeling for buried target detection, and general inverse problem methodology for proliferating populations.
Our efforts have continued on development of efficient accurate integration of Fokker Planck equations. Our objective is to analyze and optimize the dynamic behavior of nonlinear stochastic differential equations, especially the stochastic resonance effects based on the probability density function generated by the Fokker Planck equation. Our approach uses the backward characteristic method and the time-splitting integration of the Fokker Planck equation.

Status/Progress
1. *Development of innovative computational approaches for general classes of Fokker-Planck systems and uncertainty quantification:* Our efforts on developing efficient accurate integration of Fokker Planck equations have continued with success. One objective is to analyze and optimize the dynamic behavior of nonlinear stochastic differential equations, especially the stochastic resonance effects based on the probability density function generated by the Fokker Planck equation. One approach uses the backward characteristic method and the time-splitting integration of the Fokker Planck equation. We use cubic interpolation based on the solution and its derivatives to resolve the required accuracy. Then the solution and its derivatives are simultaneously updated by the backward characteristic method. We have successfully implemented a proposed time-splitting integration of the Fokker Planck equation. The method is very efficient and stable and allows one to have a large time-stepsize. We have used these methods to develop dynamic object identification and stochastic resonance techniques for detecting subliminal objects. In other efforts, we have developed fast computational methods for certain classes of Fokker Planck equations by conversion to an equivalent but simpler first order hyperbolic system with uncertainty in the coefficients which can be correctly

viewed as a random differential equation in which the coefficients are random variables. In [28] we consider this alternative approach to the use of nonlinear stochastic Markov processes (which have a Fokker-Planck or Forward Kolmogorov representation for density) in modeling dynamic propagation of uncertainty in dynamic systems (populations of particles, individuals, etc.). These alternate formulations, which involve imposing probabilistic structures on a family of deterministic dynamical systems, are shown to yield pointwise equivalent probability densities which are solutions to the corresponding Fokker-Planck equations. Moreover, these formulations lead to fast efficient calculations in inverse problems as well as in forward simulations. In our most recent efforts we derive a broad class of nonlinear stochastic formulations for which such an alternate representation is readily found. We note that all of this is part of our efforts on uncertainty quantification/propagation in inverse problems.

2. *Numerical methods for systems with discontinuous coefficients*. We have developed a new multi-moment method for one-dimensional hyperbolic equations with discontinuous coefficients. The method is based on the backward characteristic method and uses the solution and its derivative as unknowns and cubic Hermite interpolation for each computational cell. An exact update formula for solution and its derivative for variable wave speed is derived and used for efficient time integration. At points of discontinuity we develop a piecewise cubic Hermite interpolation based on interface conditions. The method is extended to the one-dimensional Maxwell equation with variable material properties. The method is fully explicit unconditionally stable, and very efficient and accurate (third order in time and space). An extension to two-dimensional case has been carried out [40]..

3. Numerical methods were developed for elastic waves propagation in a Kelvin-Voigt media. The objective is to simulate an elastic wave launched by a thumper placed on the surface of the ground. and propagates through the homogeneous Kelvin-Voigt media and interacts with a buried target in the ground. We use the time-splitting method to treat the Kelvin-Voigt damping effect. A finite difference time-domain (FDTD) scheme based on the second order Yee's scheme is used for the elastic wave equation. The Perfect matched layer (PML) method is used for the absorbing boundary treatment. Numerical results demonstrate the applicability of the proposed method and the viscous media effect. Also, the fourth order Yee's scheme is developed and tested for elastic wave propagations.

4. *Inverse problems for nonlinear delay systems.* We consider inverse or parameter estimation problems for general nonlinear nonautonomous dynamical systems with distributed and/or discrete delays. The parameters may be from a Euclidean set as usual, may be time dependent coefficients or may be probability distributions across a family of parameters as arise in aggregate data problems. New theoretical convergence results for finite dimensional approximations to the systems are given in [26]. Several examples (insect populations with time dependent maturation and death rate, cellular level HIV models with uncertainty in process delays, and models for changing behavior in response to alcohol therapy) are used to illustrate the ideas. Computational results that demonstrate efficacy of the approximations are presented for behavioral control systems.

5. *Generalized sensitivities and optimal experimental design:* In an earlier work we considered the problem of estimating model parameters θ using a weighted least squares criterion by introducing an abstract framework involving generalized measurement procedures characterized by probability measures. We take an optimal design perspective, the general premise (illustrated via examples) being that in any data collected, the information content with respect to estimating parameters may vary considerably from one time measurement to another, and in this regard some measurements may be much more informative than others. Typical optimal design methods for inverse or parameter estimation problems are designed to choose optimal sampling distributions through minimization of a specific cost function involving the Fisher Information Matrix. It is hoped that the inverse problem will produce parameter estimates with increased accuracy using data from the "optimal" sampling distribution. In recent efforts [17] we compare three different optimal design methods. These are two standard designs, *D-optimal*, *and E-optimal*, plus that proposed in our work, the *SE-optimal design*. The optimal sampling distributions from each design are used to compute and compare standard errors; the standard errors for parameters are computed using asymptotic theory and/or bootstrapping and the optimal mesh. We use examples (the logistic equation, the harmonic oscillator, and a standard vector model for glucose homeostasis) to illustrate ideas and verify that the newly developed method is competitive and in some cases can be superior to D-optimal and E-optimal formulations. These studies have been carried out with uncorrelated data and we are now pursuing these investigations with data sets in which correlation is present and must also be estimated.

6. *Dynamic evasion-interrogation games with uncertainty:* We report new progress [25], [29] on dynamic electromagnetic evasion-interrogation games in which the evader can use ferroelectric material coatings to attempt to avoid detection while the interrogator can manipulate the interrogating frequencies to enhance detection. The resulting problem is formulated as a two-player zero-sum dynamic differential game in which the cost functional is based on the expected value of the intensity of the reflected signal. In [29] we show that there exists a saddle point for the relaxed form of this dynamic differential game in which the relaxed controls appear bilinearly in the dynamics governed by a partial differential equation. We also have developed a computational framework for construction of approximate saddle point strategies in feedback form for a special case of this relaxed differential game with strategies and payoff in the sense of Berkovitz.
In one version of these problems, each player must incorporate significant uncertainty into their design strategies to disguise their intension and confuse their opponent. In [25], the evader is allowed to make dynamic changes to his strategies in response to the dynamic input with uncertainty from the interrogator. The problem is formulated in two different ways; one is based on the evolution of the probability density function of the intensity of reflected signal and leads to a controlled forward Kolmogorov or Fokker-Planck equation. The other formulation is based on the evolution of expected value of the intensity of reflected signal and leads to controlled backward Kolmogorov equations. A number of numerical results are developed to illustrate the usefulness of the proposed approach in exploring problems of control in a general dynamic game setting.

7. Over the past three decades there has been interest in using Pad\'e approximants K as ``reduced-order models'' for the transfer function G of a linear system. The attractive feature of this approach is that by matching the moments of G one can reproduce the steady-state behavior of G by the steady-state behavior of K, for certain classes of inputs. Indeed, in [35] we illustrate this by finding a first-order model matching a fixed set of moments for G, the causal inverse of a heat equation. A key feature of this example is that the heat equation is a minimum phase system, so that its inverse system has a stable transfer function G and that K can also be chosen to be stable. On the other hand, elementary examples show that both stability and instability can occur in reduced order models of a stable system obtained by matching moments using Pad\'e approximants and, in the absence of stability, it does not make much sense to talk about steady-state responses nor does it make sense to match moments. In [35], we have reviewed Pad\'e approximants, and their intimate relationship to continued fractions and Riccati equations, in a historical context that underscores why Pad\'e approximation, as useful as it is, is not an approximation in any sense that reflects stability.

8. The paper [39] is concerned with a problem of tracking regulation for a one dimensional Kuramoto-Sivashinsky equation. The objective in this work is to design dynamic and static controllers to drive the state of the plant at the ends of the spatial domain to desired reference signals which may be time dependent. The particular case of tracking constant reference signals is referred to as the set point control problem. To solve our static and dynamic tracking problems we employ the zero dynamics inverse design methodology recently developed to solve a variety of tracking and disturbance rejection problems for linear and nonlinear systems. We also present two numerical examples which illustrate our theoretical results.

9. In [33] we use the zero dynamics inverse (ZDI) design methodology for designing a feedback compensation scheme achieving asymptotic regulation in the case when the measured variable is required to track a reference signal w. Following the nonequilibrium formulation of output regulation we formulate the problem of asymptotic regulation by requiring zero steady-state error together with ultimate boundedness of the state of the system and the controller(s), with a bound determined by bounds on the norms of the initial data and w. Since a controller solving this problem depends only on a bound on the norm of w not on the particular choice of w, this formulation is in sharp contrast to both exact tracking, asymptotic tracking or dynamic inversion of a completely known trajectory and to output regulation with a known exosystem.

The ZDI design consists of the interconnection, via a memoryless filter, of a stabilizing feedback compensator and a cascade controller, designed in a simple, universal way from the zero dynamics of the closed-loop feedback system. This design philosophy is illustrated with a problem of asymptotic regulation for a boundary controlled viscous Burgers' equation, for which we prove that the ``zero dynamics inverse'' is input-to-state stable (ISS). In infinite dimensions, however, ISS compactness arguments are supplanted by smoothing arguments to accommodate crucial technical details, including the global existence, uniqueness and regularity of solutions to the interconnected systems.

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AFRL Point of Contact and Possible Transitions
Inverse problem methodology in the presence of uncertainty: Early efforts on this topic have been in collaboration with scientists at AFRL led by Dr. R.A. Albanese (Tel: 210-536-5710). During the past year we have exchanged visits with scientists from AFRL at WPAFB (point of contact: Jeremy Knopp, Tel: 937-255-9797) and are in the planning stages to collaborate on Nondestructive Evaluation (NDE) efforts to quantify uncertainty in detection and characterization of damage in probability of detection (POD) algorithms.
Electromagnetic interrogation of dispersive media: Problem formulation and method development involves close collaboration with Dr. Richard Albanese (Tel: 210-536-5710) and colleagues at AFRL, Brooks City Base, San Antonio, TX. The efforts on counter-interrogation and counter counter-interrogation are in direct response to questions raised by Dr. R. Albanese and his team at AFRL. We also have had lengthy phone conversations (several per year) with Albanese which have provided major influence on the direction of our efforts.

Honors/Awards
H.T. Banks, AAAS Fellow, December, 2010, SIAM Fellow, July, 2009.
H.T. Banks, S. Hu, G.M. Kepler: 2010 Lord Robert May Best Paper Prize (best paper in 2007/2008 volumes of J. Biological Dynamics) (paper co-authored with M. Davidian, and E. Rosenberg), awarded July, 2010.

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[1] H.T. Banks, A. Cintron-Arias, A. Capaldi and A. L. Lloyd, A sensitivity matrix based methodology for inverse problem formulation, J. Inverse and Ill-posed Problems, 17 (2009), 545--564.
[2] H.T. Banks, J.L. Davis and S.Hu, A computational comparison of alternatives to including uncertainty in structured population models, CRSC-TR09-14, June, 2009; in

Three Decades of Progress in Systems and Control, (X. Hu, et al., eds.) Springer, New York, 2010, 19--33.
[3] H.T. Banks, S. Hu and W.C. Thompson, Electromagnetic interrogation and the Doppler shift using the method of mappings, Mathematical and Computer Modeling, 51 (2010), 389--399.
[4] K. Ito and K. Kunisch, Asymptotic properties of feedback solutions for a class of quantum control problems, SIAM J. Control Optimiz. 48 (2009), 2323--2343.
[5] H.T. Banks, K.L. Rehm and K.L. Sutton, Dynamic social network models incorporating stochasticity and delays, CRSC-TR09-11, May, 2009; Quarterly Applied Math., 68 (2010), 783--802.
[6] H.T. Banks, K. Holm and D. Robbins, Standard error computations for uncertainty quantification in inverse problems: Asymptotic theory vs. bootstrapping, CRSC-TR09-13, June, 2009; Revised August, 2009; Revised, May, 2010; Mathematical and Computer Modeling, 52 (2010), 1610--1625.
[8] H.T. Banks, S. Hu, Z.R. Kenz and H.T. Tran, A comparison of nonlinear filtering approaches in the context of an HIV model, CRSC-TR09-16, July, 2009; Math. Biosci. Engr., 7 (2010), 213--235.
[9] H. T. Banks, W. Clayton Thompson, Karyn L. Sutton, Gennady Bocharov, Dirk Roose, Tim Schenkel, and Andreas Meyerhans Estimation of cell proliferation dynamics using CFSE data, (with), CRSC-TR09-17, August, 2009; Bull. Math. Biology, 73 (2011), 116--150.
[10] H. T. Banks, Sava Dediu, Stacey L. Ernstberger and Franz Kappel, Generalized sensitivities and optimal experimental design, CRSC-TR08-20 (Revised), November, 2009; J. Inverse and Ill-posed Problems, 18 (2010), 25--83.
[11] H.T. Banks, S. Hu and W.C. Thompson, Electromagnetic interrogation and the Doppler shift using the method of mappings, CRSC-TR09-20, December, 2009; Mathematical and Computer Modeling, 51 (2010), 389--399.
[12] J. David, H.T. Tran and H.T. Banks, Receding horizon control of HIV, CRSC-TR09-21, N.C. State University, December, 2009; Optimal Control, Applications and Methods, 32 (2011), 681--699.
[13] H.T. Banks, A. Cintron-Arias and F. Kappel, Parameter selection methods in inverse problem formulation, CRSC-TR10-03, N.C. State University, February, 2010. in Mathematical Model Development and Validation in Physiology: Application to the Cardiovascular and Respiratory Systems, Lecture Notes in Mathematics, Mathematical Biosciences Subseries, Springer-Verlag, 2011.
[14] H.T. Banks, D. Cioranescu, A.K. Criner, and W.P. Winfree, Modeling the flash-heat experiment on porous domains, CRSC-TR10-06, N.C. State University, May, 2010; Quarterly Applied Math., S 0033-569X(2011)01230-8 (15pages).
[15] A. R. Ortiz, H. T. Banks, C. Castillo-Chavez, G. Chowell, and X. Wang, A deterministic methodology for estimation of parameters in dynamic Markov chain Models, CRSC-TR10-07, N.C. State University, May, 2010; J. Biological Systems, 19 (2011), 71--100. .
[16] H.T. Banks, S. Hu and Zackary R. Kenz, A brief review of elasticity and viscoelasticity, CRSC-TR10-08, N.C. State University, May, 2010; Advances in Applied Mathematics and Mechanics, 3 (2011), 1--51.

[17] H.T. Banks, K. Holm and F. Kappel, Comparison of optimal design methods in inverse problems, CRSC-TR10-11, N.C. State University, July, 2010; Inverse Problems, 27 (2011), 075002.
[19] K. Ito and K. Kunisch, Asymptotic properties of feedback solutions for a class of quantum control problems, SIAM J. Control Optim., (2009) 2323--2343.
[20] K.Ito, K. Kunisch and Q. Zhang, Nonlinear feedback solutions for a class of quantum control problems, Optimal control of coupled systems of partial differential equations, Internat. Ser.Numer.,Math.,158, Birkhäuser Verlag, Basel, (2009), 155-170.
[21] H.T. Banks, F. Charles, M. Doumic-Jauffret, K.L. Sutton and W.C. Thompson, Label structured cell proliferation models, Applied Math Letters, 23 (2010), 1412--1415.
[22] A.R. Ortiz, H.T. Banks, C. Castillo-Chavez, G. Chowell and X. Wang, A discrete events delay differential system model for transmission of Vancomycin-resistant Enterococcus (VRE) in hospitals, J. Inverse and Ill-posed Problems, 18 (2011), 787--821.
[23] K. Ito and K. Kunisch, Semi-smooth Newton methods for time-optimal control for a class of ODEs, SIAM J. Control Optim., 48 (2010), 3997--4013.
[24] K. Ito, K. Kunisch and Q. Zhang , Feedback solution and receding horizon control synthesis for a class of quantum control problems, Applied and Numerical Partial Differential Equations, Comput. Methods Appl. Sci., 15, Springer, New York, (2010), 113--126.
[25] H.T. Banks, S. Hu, K. Ito and S.G. Muccio, Dynamic electromagnetic evasion-pursuit games with uncertainty, CRSC-TR10-13, N.C. State University, Raleigh, NC, August, 2010; Numerical Mathematics: Theory, Methods and Applications, 4 (2011), 399--418.
[26] H.T. Banks, K. Rehm and K. Sutton, Inverse problems for nonlinear delay systems, CRSC-TR10-17, N.C. State University, Raleigh, NC, November, 2010; Methods and Applications of Analysis, 17 (2010), 331--356.
[27] H.T. Banks, Taesoo Jang and Hee-Dae Kwon, Feedback control of HIV antiviral therapy with long measurement time, CRSC-TR11 -01, N.C. State University, Raleigh, NC, January, 2011; Intl. J. Pure and Applied Math, 66 (2011), 461--485.
[28] H.T. Banks and S. Hu, Nonlinear stochastic Markov processes and modeling uncertainty in populations, CRSC-TR11-02, N.C. State University, Raleigh, NC, January, 2011; Math. Biosci. Engr., 9 (2012), 1--25.
[29] H.T. Banks and S. Hu, A zero-sum electromagnetic evader-interrogator differential game with uncertainty, CRSC-TR11-04, N.C. State University, Raleigh, NC, February, 2011; Applicable Analysis, to appear.
[30] H.T. Banks, D. Cioranescu, A.K. Criner, and W.P. Winfree, Parameter estimation for the heat equation on perforated domains, CRSC-TR11-06, N.C. State University, Raleigh, NC, April, 2011; J. Inverse and Ill-posed Problems, DOI.10.1515/JIIP.2011.051 (33 pages).
[31] H.T. Banks and Shuhua Hu, Propagation of uncertainty in dynamical systems, CRSC-TR11-11, N.C. State University, Raleigh, NC, October, 2011; Proceedings 43rd ISCIE International Symposium on Stochastic Systems Theory and Its Applications, Shiga, Japan, October 28-29, 2011.

[32] H.T. Banks, K.J. Holm and F. Kappel, A Monte Carlo based analysis of optimal design criteria, CRSC-TR11-14, N.C. State University, Raleigh, NC, November, 2011; J. Inverse and Ill-posed Problems, to appear.
[33] C.I. Byrnes, D. Gilliam, C. Hu and V. Shubov, Asymptotic regulation for distributed parameter systems via zero dynamics inverse design, International Journal of Robust And Nonlinear Control, to appear.
[34] D. Gilliam, A. v. Rooij, F. Ruymgaart, Fourier-Cesaro approximation in the Hausdorff metric with application to noisy deconvolution, Introduction to Wavelets, Potsdam, New York: Sampling Publishing, (2011), pp. 304-329.
[35] C.I. Byrnes and A. Lindquist, On the stability and instability of Pad\'e approximants, Perspectives In Mathematical System Theory, Control, And Signal Processing, Lecture Notes in Control and Information Sciences, 2010, Volume 398/2010, 165-175.
[36] C. I. Byrnes, Topological methods for nonlinear oscillations, Notices of the Amer. Math. Soc., 57 (2010), 1080--1091.
[37] C.I. Byrnes, On the level and sublevel sets of a Lyapunov function, J. of Diff. Eqns., to appear.
[38] C. I. Byrnes, On the topology of Lyapunov functions for dissipative periodic processes, Emergent Problems in Nonlinear Systems and Control, Springer-Verlag, to appear.
[39] C. I. Byrnes, D.S. Gilliam, C. Hu and V.I. Shubov, Zero dynamics boundary control for regulation of the Kuramoto-Sivashinsky equation, Mathematical and Computer Modelling, Volume 52, (2010), 875-891.
[40] K.Ito and T. Takeuchi, A multi-moment scheme for the two dimensional Maxwell's equations, Wave Motion, 2011, submitted.